MATHEMATICS SPECIALIST 3,4
TEST 3 SECTION ONE 2016
NON Calculator Section
Chapters 4,5

CHURCHLANDS SENIOR HIGH SCHOOL

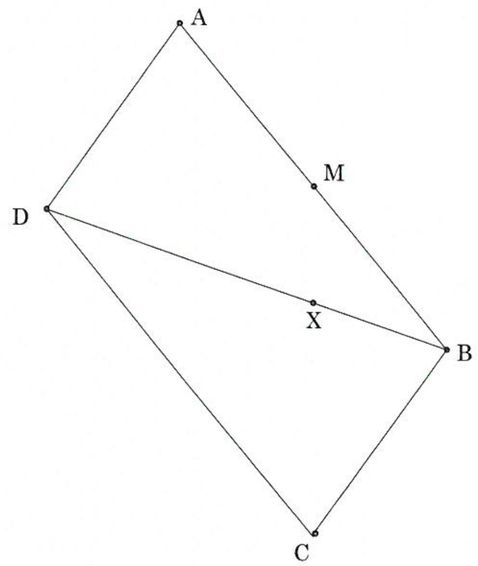
Name_______________________________

Time: 15minutes
Total: 13 marks

Question 1 **[5 marks]**

The diagram to the right shows parallelogram ABCD where $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$.

Point X divides DB internally in the ratio 2:1.
Point M is the midpoint of AB.



a) Show that $\overrightarrow{DX} = \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$. [1]

b) Find $\overrightarrow{CX}$ in terms of **a** and **b**. [1]

c) Prove that points M, X and C are collinear. [3]

Question 2 **[6 marks]**

Given the vectors, **a** = 2**i** – 3**j** + **k**, **b** = 4**j** – **k** and **c** = **i** – 2**j** – 3**k**, find:

a) 3**b** – **a** [1]

b) |**c**| [1]

c) the vector equation of the line passing through the point with position vector 3**b** and the point with position vector **a**. [2]

d) the vector equation of the plane passing through the point with position vector **b** and normal to the vector **c**. [2]

Question 3 **[2 marks]**

Find $\underline{a} \times \underline{b}$ given that $\underline{a} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

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Name______________________________

Time: 40 minutes
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Question 1 **[8 marks]**

Points A and B have co-ordinates (2, 6,-2) and (5, 0, 7) respectively.

a) Determine in parametric form the equation of the line L1 that passes through points A and B.

[2]

b) Plane P has equation $r \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 14$. Determine the co-ordinates of point C, the intersection of the line and the plane.

[2]

c) Determine to the nearest degree the acute angle between the line and the plane.

[2]

d) Calculate $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

[1]

e) Hence determine a vector equation of the line L2 parallel to plane P that passes through point A.

[1]

Question 2 **(5 marks)**

Two pilots (Abu and Jimmy) are manoeuvring their light planes into holding patterns near Jandakot airport. The planes have the following position and velocity vectors (at time t = 0 seconds) :

rA = (500i + 300j + 200k) m   vA = (–18i – 13j + 12k) m/sec

rJ = (150i – 820j + 610k) m   vJ = (–20i + 72j – 12k) m/sec

Round answers to this question to 3 significant figures where appropriate.

a) Determine the speed of Abu's plane.

[1]

b) At what angle is Jimmy's plane descending?

[2]

c) How far apart are the two planes at time t = 10 s ?

[2]

c) Determine to the nearest degree the acute angle between the line and the plane.

[2]

d) Calculate $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

[1]

e) Hence determine a vector equation of the line L2 parallel to plane P that passes through point A.

[1]

Question 3

(2 marks)

At time t minutes, the position vector of object A is given by $\underline{r}_A = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

The surface of a wall Π, has equation $\underline{r} \bullet \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = 10$. The point B with position vector $\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ lies on this wall.

a) Show A that will never hit the wall.

[2]

Question 4 **(8 marks)**

Consider the points A (3, 2, 5), B(5, 1, 8), C(5, 4, 6), D(3, 5, 3) and R(x, y, z) with position vectors **a, b, c, d** and **r** respectively.

The equation of the plane ABC is $\left(\overrightarrow{AR}\right)\bullet\left(\left(\overrightarrow{AB}\right)\times\left(\overrightarrow{AC}\right)\right)=0$

a) Determine the equation of the plane in the form $ax+by+cz+d=0$ by using the formula above. [3]

b) Verify that the points A, B and C satisfy the equation of the plane .(sub in) [2]

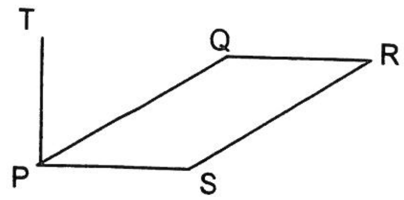
c) Explain why $(a-r)\bullet((b-a)\times(c-a))=0$ is the equation of the plane through A, B and C [3]

Question 5

(13 marks)

The points P, Q and R have position vectors $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\mathbf{q} = 4\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$ and $\mathbf{r} = 8\mathbf{i} + 21\mathbf{j} - 6\mathbf{k}$ respectively, relative to the origin. The point S has position vector $\mathbf{s}$ and is such that PQRS is a parallelogram.

a) Find the position vector of $\mathbf{s}$ relative to the origin. [2]



b) Calculate the lengths of PQ and QR, the size of angle PQR and hence the area of the parallelogram. [4]

c) Show that the vector $\mathbf{u} = 15\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ is perpendicular to the plane containing the parallelogram.

[3]

d) The point T with position vector $\mathbf{t} = a\mathbf{i} + b\mathbf{j} + 4\mathbf{k}$ lies on the line that is perpendicular to the plane, through P. Determine the volume of the pyramid PQRST. [4]

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Point X divides DB internally in the ratio 2:1.
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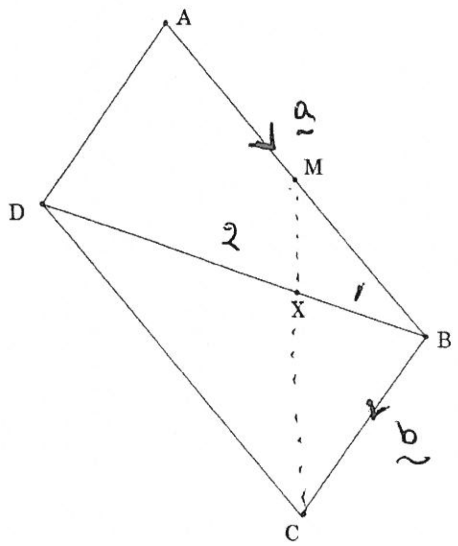

a) Show that $\overrightarrow{DX} = \frac{2}{3}\mathbf{a} - \frac{2}{3}\mathbf{b}$. [1]

$\overrightarrow{DX} = \frac{2}{3}\overrightarrow{DB}$

$= \frac{2}{3}(\overrightarrow{DA} + \overrightarrow{AB})$

$= \frac{2}{3}(-\underset{\sim}{b} + \underset{\sim}{a}) \quad \therefore \overrightarrow{DX} = \frac{2}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$

b) Find $\overrightarrow{CX}$ in terms of **a** and **b**. [1]

$\overrightarrow{CX} = \overrightarrow{CB} + \overrightarrow{BX}$

$= -\underset{\sim}{b} + \frac{1}{3}(-\underset{\sim}{a} + \underset{\sim}{b})$

$= -\frac{1}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$

c) Prove that points M, X and C are collinear. [3]

$\overrightarrow{XM}$, $\overrightarrow{CX}$ common X. {multiple of each other}

$\overrightarrow{CX} = -\frac{1}{3}\underset{\sim}{a} - \frac{2}{3}\underset{\sim}{b}$ ✓, $\overrightarrow{XM} = \overrightarrow{XB} + \overrightarrow{BM}$

$= \frac{1}{3}(\underset{\sim}{a} - \underset{\sim}{b}) + \frac{1}{2}\underset{\sim}{a}$

$= -\frac{1}{6}\underset{\sim}{a} - \frac{1}{3}\underset{\sim}{b}$ ✓

Note $2\overrightarrow{XM} = \overrightarrow{CX}$ and as X is common M, X, C are collinear ✓

Question 2 **[6 marks]**

Given the vectors, **a** = 2**i** – 3**j** + **k**, **b** = 4**j** – **k** and **c** = **i** – 2**j** – 3**k**, find:

a) 3**b** – **a** [1]

$= 3(4j - k) - (2i - 3j + k)$

$= 12j - 3k - 2i + 3j - k$

$= -2i + 15j - 4k$

b) |**c**| [1]

$= \sqrt{1^2 + 2^2 + 3^2}$

$= \sqrt{14}$

c) the vector equation of the line passing through the point with position vector 3**b** and the point with position vector **a**. [2]

$r = p + \lambda q$

$= 2i - 3j + k + \lambda(12j - 3k - (2i - 3j + k))$

$= 2i - 3j + k + \lambda(-2i + 15j - 4k)$

d) the vector equation of the plane passing through the point with position vector **b** and normal to the vector **c**. [2]

$r \cdot n = c \cdot n$

$r \cdot (i - 2j - 3k) = -5$

$(i - 2j - 3k) \cdot (4j - k)$

$= -8 + 3$

$= -5$

Question 3 **[2 marks]**

Find $\underline{a} \times \underline{b}$ given that $\underline{a} = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ and $\underline{b} = \begin{pmatrix} 3 \\ -2 \\ 2 \end{pmatrix}$

$\underline{a} \times \underline{b} = \begin{pmatrix} +2 \\ -10 \\ -13 \end{pmatrix}$

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Question 1 **[8 marks]**

Points A and B have co-ordinates (2, 6,-2) and (5, 0, 7) respectively.

a) Determine in parametric form the equation of the line L1 that passes through points A and B.

$\overrightarrow{AB} = \begin{pmatrix}5\\0\\7\end{pmatrix} - \begin{pmatrix}2\\6\\-2\end{pmatrix} = \begin{pmatrix}3\\-6\\9\end{pmatrix}$

$r = a + \lambda b$

$\therefore r = \begin{pmatrix}2\\6\\-2\end{pmatrix} + \lambda\begin{pmatrix}3\\-6\\9\end{pmatrix}$

$\therefore x = 2 + 3\lambda$
$y = 6 - 6\lambda$
$z = -2 + 9\lambda$

[2]

b) Plane P has equation $r \bullet \begin{pmatrix}1\\3\\2\end{pmatrix} = 14$. Determine the co-ordinates of point C, the intersection of the line and the plane.

$\begin{pmatrix}2+3\lambda\\6-6\lambda\\-2+9\lambda\end{pmatrix} \cdot \begin{pmatrix}1\\3\\2\end{pmatrix} = 14$

$2 + 3\lambda + 18 - 18\lambda - 4 + 18\lambda = 14$

$\Rightarrow 16 + 3\lambda = 14$

$\therefore \lambda = -\frac{2}{3}$

[2]

$\therefore$ Coordinate: $(0, 10, -8)$

c) Determine to the nearest degree the acute angle between the line and the plane.

$\underset{\sim}{a} \cdot \underset{\sim}{b} = |a||b|\cos\theta$.

$\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 3$

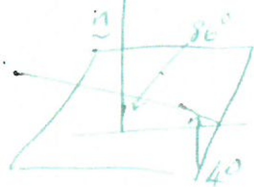
$\therefore \cos\theta = \dfrac{3}{\sqrt{14}\sqrt{126}}$

$\therefore \theta = 4^\circ$

$\overrightarrow{AB}$ is direction of the line ie $\begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$

$\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$ is normal to the plane

$\theta = 85.9 \simeq 86^\circ$.



[2]

d) Calculate $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} = 0$

[1]

e) Hence determine a vector equation of the line L2 parallel to plane P that passes through point A.

↳ ⊥ to other

$\underset{\sim}{r} = \begin{pmatrix} 2 \\ 6 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$

[1]

4

Question 2

(5 marks)

Two pilots (Abu and Jimmy) are manoeuvring their light planes into holding patterns near Jandakot airport. The planes have the following position and velocity vectors (at time t = 0 seconds) :

rA = (500i + 300j + 200k) m　　　　vA = (−18i − 13j + 12k) m/sec
rJ = (150i − 820j + 610k) m　　　　vJ = (−20i + 72j − 12k) m/sec

Round answers to this question to 3 significant figures where appropriate.

a) Determine the speed of Abu's plane.

$$\text{Speed} = \sqrt{18^2 + 13^2 + 12^2}$$
$$= 25.2 \text{ m/s}$$

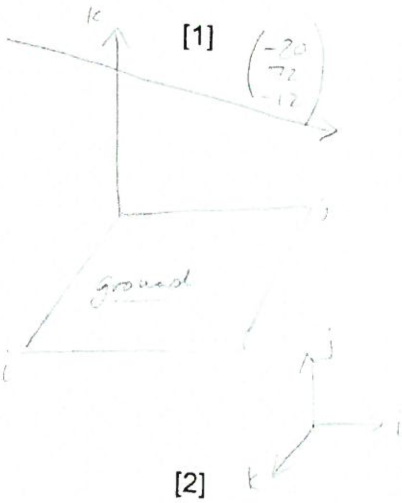
[1]

b) At what angle is Jimmy's plane descending?

$$\begin{pmatrix} -20 \\ 72 \\ -12 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = (-12) \qquad \therefore 9.12^\circ$$

$$\cos\theta = \frac{-12}{\sqrt{400 + 72^2 + 144}}$$

$$\theta = 99.12^\circ$$



[2]

c) How far apart are the two planes at time t = 10 s ?　　t = 10

$$r_A = (500i + 300j + 200k) + 10(-18i - 13j + 12k)$$

$$r_B = (150i - 820j + 610k) + 10(-20i + 72j - 12k)$$

$$\sqrt{(320 + 50)^2 + (170 + 100)^2 + (320 - 490)^2}$$

$$|r_A - r_B| = 488.56 \text{ m}$$

489m.

[2]

/5

Question 3

(2 marks)

At time t minutes, the position vector of object A is given by $\underline{r}_A = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix} + t\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$

The surface of a wall Π, has equation $\underline{r} \bullet \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = 10$. The point B with position vector $\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ lies on this wall.

a) Show A that will never hit the wall.

[2]

$\Rightarrow \begin{pmatrix} 6+t \\ -2+-2t \\ 3+t \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} = 10$ should hold

But $-12 - 2t - 2 - 2t + 12 + 4t = -2$

$\therefore -2 \neq 10$ Hence no intersection!

2

Question 4

(8 marks)

Consider the points A (3, 2, 5), B(5, 1, 8), C(5, 4, 6), D(3, 5, 3) and R(x, y, z) with position vectors **a**, **b**, **c**, **d** and **r** respectively.

The equation of the plane ABC is $(\overrightarrow{AR})\bullet((\overrightarrow{AB})\times(\overrightarrow{AC}))=0$

a) Determine the equation of the plane in the form $ax+by+cz+d=0$ by using the formula above.

[3]

$\overrightarrow{AR} = \underset{\sim}{r} - \underset{\sim}{a}$ etc $\overrightarrow{AB} = b - \underset{\sim}{a}$, $\overrightarrow{AC} = \underset{\sim}{c} - \underset{\sim}{a}$

FIRST

$\begin{pmatrix} x-3 \\ y-2 \\ z-5 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \times \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix} = 0$ ✓

$\begin{pmatrix} x-3 \\ y-2 \\ z-5 \end{pmatrix} \cdot \begin{pmatrix} -7 \\ 4 \\ 6 \end{pmatrix} = 0$ ✓

$\therefore -7x + 4y + 6z - 17 = 0$ ✓

$\therefore -7x + 21 + 4y - 8 + 6z - 30 = 0$.

b) Verify that the points A, B and C satisfy the equation of the plane .(sub in)

[2]

A ✓

B ✓

C ✓

$-7(3) + 4(5) + 6(3) - 17 = 0$ ✓

c) Explain why $(a-r)\bullet((b-a)\times(c-a))=0$ is the equation of the plane through A, B and C

[3]

$\overrightarrow{AB} \times \overrightarrow{AC} = (\underset{\sim}{b} - \underset{\sim}{a}) \times (\underset{\sim}{c} - \underset{\sim}{a})$ is ⊥ to the plane ABC

$\overrightarrow{AR} = (\underset{\sim}{a} - \underset{\sim}{r})$ is a vector in the plane

$(\underset{\sim}{a} - \underset{\sim}{r}) \cdot ((\underset{\sim}{b} - \underset{\sim}{a}) \times (\underset{\sim}{c} - \underset{\sim}{a})) = 0$ as the two vectors are ⊥

8

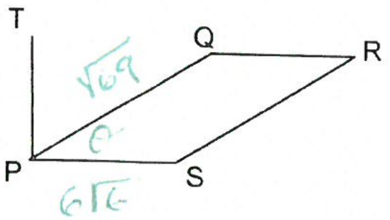
Question 5 **(13 marks)**

The points P, Q and R have position vectors $\mathbf{p} = 2\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$, $\mathbf{q} = 4\mathbf{i} + 7\mathbf{j} - 4\mathbf{k}$ and $\mathbf{r} = 8\mathbf{i} + 21\mathbf{j} - 6\mathbf{k}$ respectively, relative to the origin. The point S has position vector $\mathbf{s}$ and is such that PQRS is a parallelogram.

a) Find the position vector of $\mathbf{s}$ relative to the origin. [2]

$s = p + r - q$ $\quad \begin{pmatrix}2\\3\\3\end{pmatrix} + \begin{pmatrix}4\\14\\-2\end{pmatrix}$

$= 6i + 17j + k$



b) Calculate the lengths of PQ and QR, the size of angle PQR and hence the area of the parallelogram. [4]

$\vec{PQ} = q - p$

$= 2i + 4j - 7k$

$\therefore |\vec{PQ}| = \sqrt{69}$ ✓

$\vec{QR} = r - q$ $\quad \begin{pmatrix}2\\4\\-7\end{pmatrix} \cdot \begin{pmatrix}4\\14\\-2\end{pmatrix}$

$= 4i + 14j - 2k$

$|\vec{QR}| = 6\sqrt{6}$ ✓

$\angle PQR = 50.3^\circ$ ✓

$A = 2 \times \frac{1}{2} \times \sqrt{69} \times 6\sqrt{6} \sin 50.3$

$= 93.9 \text{ cm}^2$ ✓

4

c) Show that the vector $\mathbf{u} = 15\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$ is perpendicular to the plane containing the parallelogram. [3]

$$\underset{\sim}{u} \cdot \overrightarrow{PQ} = \begin{pmatrix} 15 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}$$

$$= 30 - 16 - 14$$

$$= 0$$

$$u \cdot \overrightarrow{QR} = \begin{pmatrix} 15 \\ -4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ -14 \\ 2 \end{pmatrix}$$

$$= 60 + 56 + 4$$

$$= 0$$

As both dot products are zero

$\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are non-parallel vectors in the plane. then $\underset{\sim}{u}$ is ⊥ to the plane

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OR $\begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} \times \begin{pmatrix} -4 \\ -14 \\ 2 \end{pmatrix} = \begin{pmatrix} 90 \\ -24 \\ 12 \end{pmatrix}$

The cross product of 2 non parallel vectors in the plane gives a vector perpendicular to the plane

$\begin{pmatrix} 90 \\ -24 \\ 12 \end{pmatrix} = 6 \begin{pmatrix} 15 \\ -4 \\ 2 \end{pmatrix}$ ∴ The normal is parallel to $\underset{\sim}{u} = 15i - 4j + 2k$

∴ $\underset{\sim}{u}$ is ⊥ to the plane.

3

d) The point T with position vector $\mathbf{t} = a\mathbf{i} + b\mathbf{j} + 4\mathbf{k}$ lies on the line that is perpendicular to the plane, through P. Determine the volume of the pyramid PQRST. [4]

T lies on the line

$$\begin{pmatrix}2\\3\\3\end{pmatrix} + \lambda\begin{pmatrix}15\\-4\\2\end{pmatrix} = \begin{pmatrix}a\\b\\4\end{pmatrix}$$

so using k coefficients

$$3 + 2\lambda = 4 \Rightarrow \lambda = 0.5$$

$$\underline{t} = 9.5\underline{i} + \underline{j} + 4\underline{k}$$

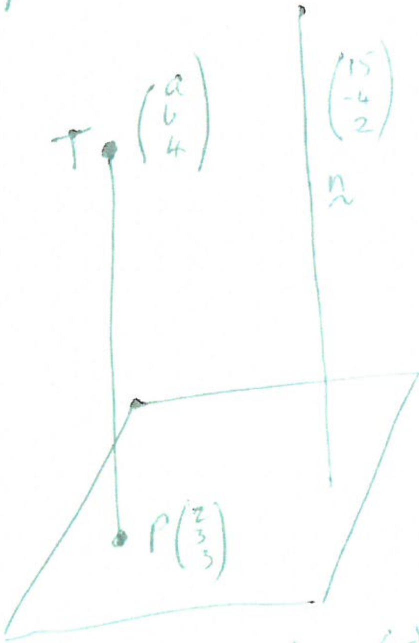
$$\overrightarrow{PT} = 7.5\underline{i} - 2\underline{j} + \underline{k}$$

$$\therefore |t| = \frac{7\sqrt{5}}{2}$$

$$V = \frac{1}{3} \times A \times h$$

$$= \frac{1}{3} \times 93.91 \times 7.826$$

$$= 245 \text{ cm}^3$$



$$PT = \begin{pmatrix}a-2\\b-3\\1\end{pmatrix} = \lambda\begin{pmatrix}15\\-4\\2\end{pmatrix}$$

$$\begin{pmatrix}15\\-4\\2\end{pmatrix} = 2\begin{pmatrix}a-2\\b-3\\1\end{pmatrix}$$

$\therefore a = 9.5$

$b = 1$

$$T = \begin{pmatrix}9.5\\1\\4\end{pmatrix}$$

$\lambda = 0.5$

$$\overrightarrow{PT} = \begin{pmatrix}7.5\\-2\\1\end{pmatrix}$$

4